SOLAR HEAT GAINS AND SKY RADIATION

Solar heat gains from windows are resulting from a global energy balance. Optical energy *transmitted* through the windows is entirely reemitted as infrared heat in the building, while optical energy *absorbed* by the windows is partially reemitted as infrared heat in the building, the other part being reemitted as infrared heat to the outdoor. The total reemitted heat in the building, divided by the whole sun optical energy reaching the window, is defined as the *window solar factor*.

Solar heat gains are computed as the sum of direct, diffuse and reflected components (ref. [19], [23]).

2.1. Direct solar heat gains

Direct solar heat gains through windows are computed through:

$$q_{b,w} = \sum_{slopes}\left(\sum_{orientations} SF.I_b.A_w\right) \qquad SF = (1 - f_{fr})g \tag{2.1}$$

$q_{b,w}$: Heat gains from direct solar intensity through windows *W*
SF : Window solar factor -
g : Glazing solar factor -
f_{fr} : Ratio of frame area in the whole window area -
I_b : Direct solar intensity on a plane of a given slope and azimuth W/m^2
A_w : Window area m^2

The solar intensity reaching a plane of a given slope and azimuth is related to the solar intensity reaching a horizontal plane by:

$$I_b = I_{bh}.\frac{\cos\theta}{\cos\theta_z} \tag{2.2}$$

I_{bh} : Direct solar intensity measured on a horizontal plane (weather data) W/m^2
θ : Angle between sun beams and the normal direction to the given wall *rad*
θ_z : Angle between sun beams and the vertical direction *rad*

The equivalent solar area of a window is defined in order to directly compute $q_{b,w}$ from I_{bh} , which is a weather data independent from the wall azimuth and slope, instead of I_b :

$$q_{b,w} = \sum_{slopes}\left(\sum_{orientations} SF.I_{bh}.A_{eq,w}\right) \tag{2.3}$$

So:

$$A_{eq,w} = A_w.\frac{\cos\theta}{\cos\theta_z} \tag{2.4}$$

: Equivalent solar window area including glazing and frame *m^2*

The direct solar radiation reaching a window of a given orientation and slope is either equal to $A_w.I_b$ or to $A_{eq,w}.I_{bh}$.

2.1.1. Solar ratios

Values of the ratio $\cos\theta/\cos\theta_z$ corresponding to *vertical* windows can be tabulated as function of the window azimuth, for a given building location (latitude and longitude).

Values of the ratio $\cos\theta/\cos\theta_z$ corresponding to *pitched* windows are then deduced from the values computed for vertical windows, through:

$$(\cos\theta/\cos\theta_z)_{pitched} = \cos p + \sin p.(\cos\theta/\cos\theta_z)_{vertical} \tag{2.5}$$

p : Slope of the pitched window (*p=0* for horizontal position; *p=π/2* for vertical) *rad*;

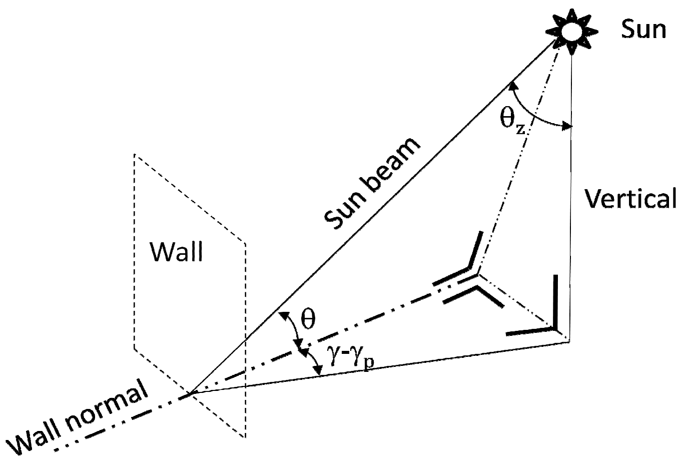

Fig. 2.1: Angles θ and θ_z as well as azimuth difference between sun and wall for a vertical wall (p=π/2).

The equations to compute the ratio $\cos\theta/\cos\theta_z$ are described below.

The angle θ between sum beams and wall normal direction can be computed from (fig. 2.1):

$$\cos\theta = \cos p.\cos\theta_z + \sin p.\sin\theta_z.\cos(\gamma - \gamma_p) \tag{2.6}$$

: Angle between sum beams and the normal direction to the given wall *rad*
p : Slope of the given wall (*p=0* for horizontal wall; *p=π/2* for vertical wall) *rad*

: Angle between sum beams and vertical direction *rad*

: *Sun* azimuth ($\gamma = 0$ when the sun is on the south; $\gamma < 0$ when the sun is on the east side and $\gamma > 0$ when the sun is on the west side) *rad*

: *Wall* azimuth ($\gamma_p = 0$ when the wall is facing the south; $\gamma_p < 0$ when the wall is facing the east side and $\gamma_p > 0$ when the wall is facing the west side) *rad*

As the wall characteristics are known, *p* and γ_p are given as data, while the sun position angles θ_z and γ, are computed hour by hour from the expressions hereunder.

The angle θ_z between sum beams and vertical direction can be computed from (fig. 2.2):

$$\cos\theta_z = \sin\delta.\sin\varphi + \cos\delta.\cos\varphi.\cos\omega \tag{2.7}$$

: Sun declination *rad*
: Latitude *rad*
: True solar time *rad*

The sun azimuth can be computed from (fig. 2.2):

$$\sin\gamma = \frac{\cos\delta.\sin\omega}{\sin\theta_z} \tag{2.8}$$

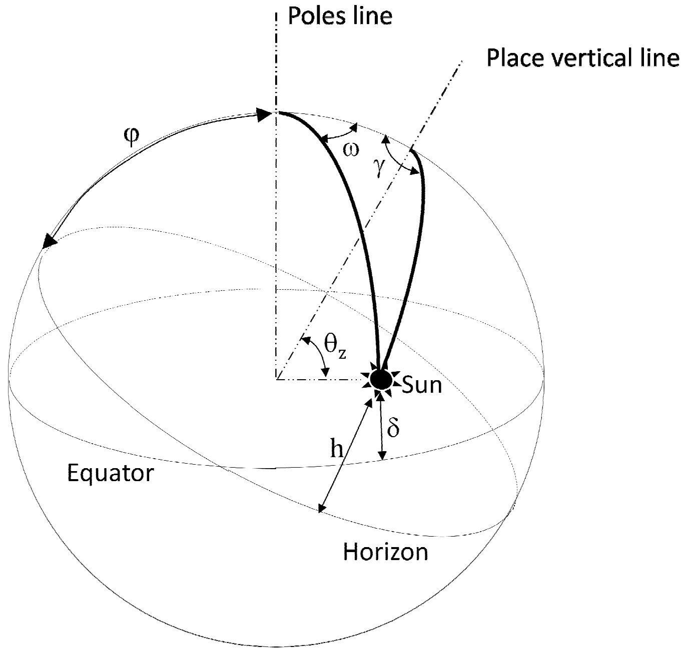

Fig. 2.2: Angle θ_z as function of sun declination δ, latitude φ and true solar time ω

The data needed to compute the sun position angles θ_z and γ (expressions (2.7) and (2.8)) are given in §2.1.2. Once those angles are computed hour by hour, the solar ratio $\cos\theta/\cos\theta_z$ can also be computed hour by hour for the whole year. Fig. 2.3, 2.4 and 2.5 give the results corresponding to vertical walls for summer solstice, spring equinox and winter solstice.

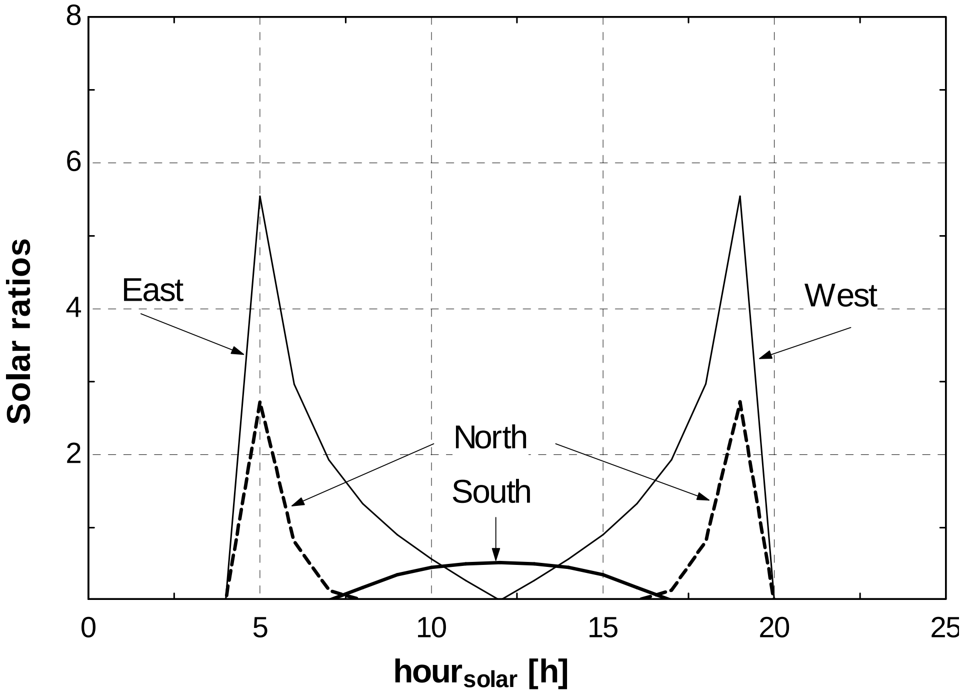

Fig. 2.3: Solar ratios for vertical South, East and West oriented walls (full lines) and for vertical North oriented wall (dotted line) on the 21st of June.

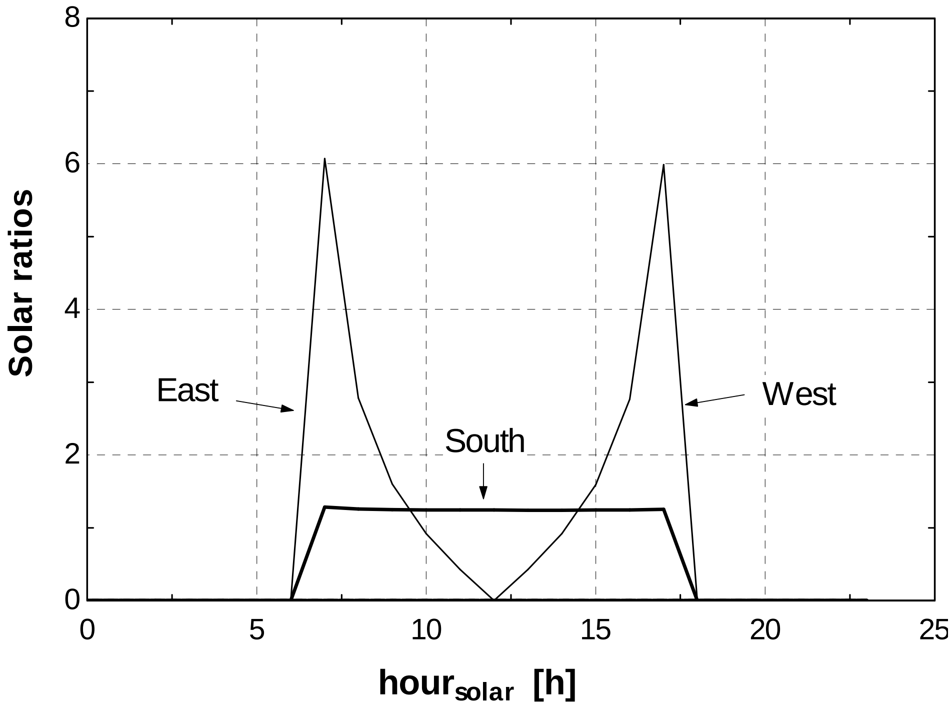

Fig. 2.4: Solar ratios for vertical South, East and West oriented walls (full lines) on the 21st of March.

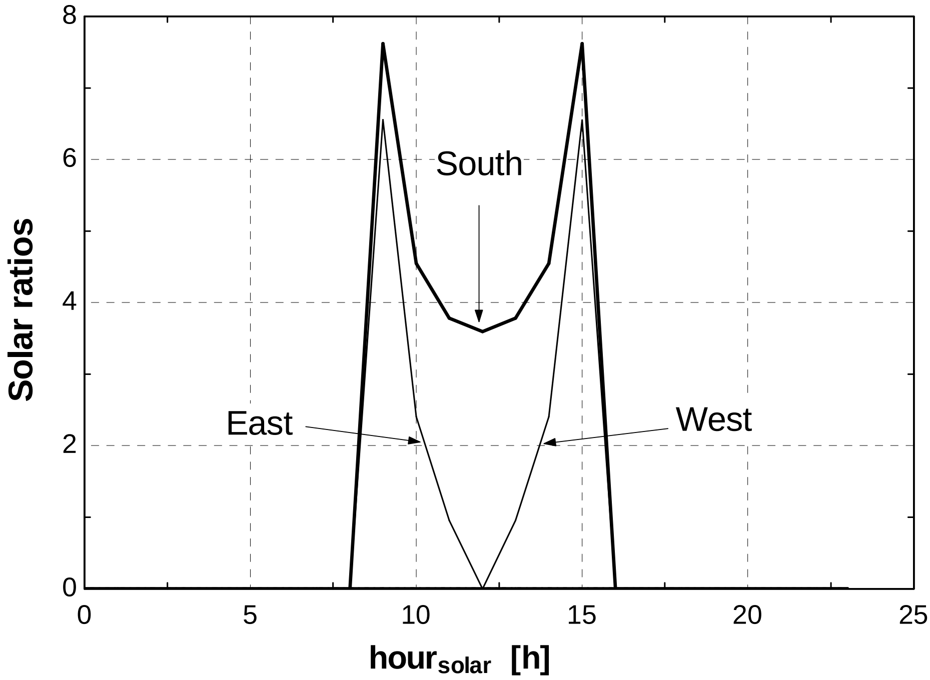

Fig. 2.5: Solar ratios for vertical South, East and West oriented walls (full lines) on the 21st of December.

2.1.2. Sun position angles

The computation of the sun position angles θ_z and γ(equations (2.7) and (2.8)) requires the knowledge of the building location *latitude* φ and *longitude* λ, of the *sun declination* δ and of the *true solar hour* ω (fig. 2.2).

The building location *latitude* φ is given as data and must be expressed in *radiant*.

The *sun declination* δ is computed by (ref. [20], [24]):

$$\delta = 0{,}40928.\sin\left(2.\pi.\left(\frac{284+j}{365}\right)\right) \tag{2.9}$$

δ : Sun declination *rad*
j : Number of the day in the year *day*

The *true solar time* ω, expressed in radiant, must be translated in hour, as ω_h , in order to be related to *UTC (Coordinated Universal Time)*:

$$\omega_h = \omega.\frac{\pi}{12} \tag{2.10}$$

ω : True solar time expressed in *rad* ω_h : True solar time expressed in *h*

$$\omega_h = UTC - 12 - \lambda_h + ET \qquad (2.11)$$

ω_h : True solar time expressed in *h*
λ_h : Longitude expressed in *h*
UTC : Coordinated Universal Time in *h*
ET : Equation of Time in *h*

The longitude λ_h must be expressed in *hour*, from the value of the longitude λ expressed in *radiant*:

$$\lambda_h = \lambda . \frac{\pi}{12} \qquad (2.12)$$

The equation of time *ET* accounts for the small perturbations due to the combination of the tilt of the Earth's rotation axis and the eccentricity of its orbit. So the time read from a sun dial runs ahead of clock time by 16 min 33s in October 31-November 1, or falls behind by 14 min 6 s around February 11-12. The equation of time *ET* is given by (ref. [20], [24]):

$$ET = (1/60).\,(- 0{,}00037 + 0{,}43177.\cos(\beta.j) - 3{,}165.\cos(2.\,\beta.j) - 0{,}07272.\cos(3.\,\beta.j) - 7{,}3764.\sin(\beta.j) - 9{,}3893.\sin(2.\,\beta.j) - 0{,}24498.\sin(3.\,\beta.j)) \qquad (2.13)$$

ET : Equation of Time in *h*
β : Conversion factor $\beta = 2.\pi / 365$ expressed in *rad/day*
j : Number of the day in the year *day*

The Coordinated Universal Time *UTC* equals *12.00 Z* when the sun is at its highest point in the sky, on the Greenwich meridian, for the day under consideration. It can be related to clock time in winter and in summer, by expressions depending on the place under consideration. For Belgium:

$$h_{cw} = UTC + 1 \qquad h_{cs} = UTC + 2 \qquad (2.14)$$

h_{cw} : Clock time in winter *h*
h_{cs} : Clock time in summer *h*
UTC : Coordinated Universal Time in *h*

The Coordinated Universal Time *UTC* can also be related to the *solar time* h_{solar} , defined specifically for a given place, though that h_{solar} equals *12h00* when the sun is at its highest point in the sky, at that location, for the day under consideration:

$$h_{solar} = UTC - \lambda_h + ET \qquad (2.15)$$

h_{solar} : Solar time of the place under consideration *h*
UTC : Coordinated Universal Time in *h*
λ_h : Longitude expressed in *h*
ET : Equation of Time in *h*

2.2. Diffuse and reflected solar heat gains

Heat gains due to diffuse and reflected solar intensities through windows are computed through:

$$q_{dr,w} = \sum_{slopes} \left(SF.I_{dr}. \sum_{orientations} A_w \right) \qquad SF = (1 - f_{fr})g \qquad (2.16)$$

$q_{dr,w}$: Heat gains through windows, from diffuse and reflected solar intensities *W*
SF : Window solar factor -
g : Glazing solar factor -
f_{fr} : Ratio of frame area in the whole window area -
I_{dr} : Diffuse and reflected solar intensities on a plane of a given slope *W/m²*
A_w : Window area including glazing and frame *m²*

If the diffuse and reflected solar intensities are considered as isotropic:

$$I_{dr} = \frac{(1+\cos p)}{2} I_{dh} + \frac{(1-\cos p)}{2} .\rho.I_{th} \qquad I_{th} = I_{dh} + I_{bh} \qquad (2.17)$$

I_{dh} : Diffuse solar intensity measured on a horizontal plane (weather data) *W/m²*
I_{bh} : Direct solar intensity measured on a horizontal plane (weather data) *W/m²*
I_{bh} : Total solar intensity measured on a horizontal plane (weather data) *W/m²*
ρ : Surrounding ground albedo -

2.3. Sky radiation

The infrared radiation emitted to the sky by an area whose slope is *p* with the horizontal plane, can be expressed as function of the infrared radiation emitted by an horizontal plane as follows, supposing equal sky clearness conditions:

$$I_{ir} = I_{ir,h} . \frac{(1 + \cos p)}{2} \tag{2.18}$$

p : Slope of the wall with the horizontal plane
I_{ir} : Infrared radiation emitted by an area of slope *p* *W/m²*

Expression (2.17) gives the infrared radiation I_{ir} emitted by a slope *p* wall, for a given time and place, provided the infrared radiation $I_{ir,h}$ of a horizontal plane at the same time and place, are known. Those horizontal plane radiations are computed as function of the sky clearness *J*:

$$I_{ir,h} = I_{ir,h,cc} + \frac{J - J_{cc}}{J_{cs} - J_{cc}} . (I_{ir,h,cs} - I_{ir,h,cc}) \tag{2.19}$$

$I_{ir,h}$: Infrared radiation emitted by an horizontal plane *W/m²*
$I_{ir,h,cs}$: Infrared radiation of a horizontal plane for clear sky conditions *W/m²*
$I_{ir,h,cc}$: Infrared radiation of a horizontal plane for covered sky conditions *W/m²*

The following values are usually considered [36]:

$I_{ir,h,cs} \approx 100 \ W/m^2$ and $IR_{ir,h,cc} \approx 45 \ W/m^2$

J : Relative solar intensity at the given time,
J_{cc} : Relative solar intensity for covered sky conditions, with J_{cc} *=0,354*
J_{cs} : Relative solar intensity for clear sky conditions, with J_{cs} *=1*

The relative solar intensity *J* is computed from the total solar intensity on a horizontal plane:

$$J = \frac{I_{t,h}}{I_{t,h,cs}} \tag{2.20}$$

$I_{t,h}$: Total solar intensity on a horizontal plane at a given time *W/m²*
$I_{t,h,cs}$: Total solar intensity on a horizontal plane for clear sky conditions *W/m²*