APPENDIX

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A. Basics of morphological transformations

From References [1], [2]

*Morphological transformations* extract and alter the structure of particles in an image. They fall into two categories:

- *Binary Morphology* functions, which apply to binary images
- *Grayscale morphology* functions, which apply to gray-level images

In grayscale morphology, a pixel is compared to those pixels surrounding it in order to keep the pixels whose values are the smallest (in the case of an erosion) or the largest (in the case of a dilation).

**Understanding Dilation and Erosion**

Morphology is a broad set of image processing operations that process images based on shapes. Morphological operations apply a structuring element to an input image, creating an output image of the same size. In a morphological operation, the value of each pixel in the output image is based on a comparison of the corresponding pixel in the input image with its neighbors. By choosing the size and shape of the neighborhood, you can construct a morphological operation that is sensitive to specific shapes in the input image. Morphological functions position the origin of the structuring element, its center element, over the pixel of interest in the input image.

The most basic morphological operations are dilation and erosion. Dilation adds pixels to the boundaries of objects in an image, while erosion removes pixels on object boundaries. The number of pixels added or removed from the objects in an image depends on the size and shape of the structuring element used to process the image. In the morphological dilation and erosion operations, the state of any given pixel in the output image is determined by applying a rule to the corresponding pixel and its neighbors in the input image. The rule used to process the pixels defines the operation as a dilation or an erosion. The following table lists the rules for both dilation and erosion.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Rule</th>
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<tbody>
<tr>
<td>Dilation</td>
<td>The value of the output pixel is the <em>maximum</em> value of all the pixels in the input pixel’s neighborhood. In a binary image, if any of the pixels is set to the value 1, the output pixel is set to 1.</td>
</tr>
<tr>
<td>Erosion</td>
<td>The value of the output pixel is the <em>minimum</em> value of all the pixels in the input pixel’s neighborhood. In a binary image, if any of the pixels is set to 0, the output pixel is set to 0.</td>
</tr>
</tbody>
</table>

Dilation and erosion are often used in combination to implement image processing operations. For example, the definition of a morphological *opening* of an image is an erosion followed by a dilation, using the same structuring element for both operations.
The related operation, morphological closing of an image, is the reverse: it consists of dilation followed by an erosion with the same structuring element.

Figure A - 1 illustrates the dilation of a binary image. Note how the structuring element defines the neighborhood of the pixel of interest, which is circled. The dilation function applies the appropriate rule to the pixels in the neighborhood and assigns a value to the corresponding pixel in the output image. In Figure A - 1, the morphological dilation function sets the value of the output pixel to 1 because one of the elements in the neighborhood defined by the structuring element is on.

![Morphological Dilation of a Binary Image](image)

Let’s illustrate (Figure A - 2) the results of binary morphology by applying a disk-shaped structuring element on a square-shaped image (dark-blue).

<table>
<thead>
<tr>
<th>Erosion</th>
<th>Dilatation</th>
<th>Opening</th>
<th>Closing</th>
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<tbody>
<tr>
<td><img src="image" alt="Erosion Example" /></td>
<td><img src="image" alt="Dilatation Example" /></td>
<td><img src="image" alt="Opening Example" /></td>
<td><img src="image" alt="Closing Example" /></td>
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</table>

The erosion of the dark-blue square by a disk, resulting in the light-blue square. The dilation of the dark-blue square by a disk, resulting in the light-blue square with rounded corners. The opening of the dark-blue square by a disk, resulting in the light-blue square with round corners. The closing of the dark-blue shape (union of two squares) by a disk, resulting in the union of the dark-blue shape and the light-blue areas.

The gray-level morphology functions apply to gray-level images. You can use these functions to alter the shape of regions by expanding bright areas at the expense of dark areas and vice versa. These functions smooth gradually varying patterns and increase the contrast in boundary areas.

Figure A - 3 illustrates the dilatation processing for a grayscale image. The figure shows the processing of a particular pixel in the input image. Note how the function applies the
rule to the input pixel’s neighborhood and uses the highest value of all the pixels in the neighborhood as the value of the corresponding pixel in the output image.

Figure A - 3
Morphological Dilation of a Grayscale Image

A gray-level erosion reduces the brightness of pixels that are surrounded by neighbors with a lower intensity. The neighborhood is defined by a structuring element. A gray-level dilation increases the brightness of each pixel that is surrounded by neighbors with a higher intensity. The neighborhood is defined by a structuring element. The gray-level dilation has the opposite effect of the gray-level erosion because dilating bright regions also erodes dark regions.

The gray-level opening function consists of a gray-level erosion followed by a gray-level dilation. It removes bright spots isolated in dark regions and smooths boundaries. The effects of the function are moderated by the configuration of the structuring element. This operation does not significantly alter the area and shape of particles because erosion and dilation are morphological opposites. Bright borders reduced by the erosion are restored by the dilation. However, small bright particles that vanish during the erosion do not reappear after the dilation.

The gray-level closing function consists of a gray-level dilation followed by a gray-level erosion. It removes dark spots isolated in bright regions and smooths boundaries. The effects of the function are moderated by the configuration of the structuring element. This operation does not significantly alter the area and shape of particles because dilation and erosion are morphological opposites. Bright borders expanded by the dilation are reduced by the erosion. However, small dark particles that vanish during the dilation do not reappear after the erosion.
APPENDIX B
Matlab Script File

B. Matlab Script File

The following script presents the main operations of the procedure, which are related to the preprocessing steps described in Chapter II - Section 3.2.

-------------------------------------------------------------------------------

Clear all;
imax=14

% Used structuring elements
se=strel('disk',3);
se1=strel('disk',3);
se2=strel('disk',1);

for i=22:imax
file = sprintf('ACP12B%d.TIF',i);
I1 = imread(file);

% Crop the image
l1=I1(1:423,1:712);

% Evaluate background
background = imopen(l1,strel('disk',10));

% Subtract background
I2 = imsubtract(I1,background);

% Erosion
erodedBW =imerode(I2,se1);

% Contrast Enhancement
J = imsubtract(imadd(erodedBW,imtophat(erodedBW,se)),imbothat(erodedBW,se));

% Binarization
BW4= im2bw(J,graythresh(J));

% Removal of bridges
final = imopen(BW4,se2);

% Removal of impulsive noise
BW=medfilt2(BW4,[3,3]);

-------------------------------------------------------------------------------

- 192 -
% Coverage rate
blackCount = sum(sum(final == 0));
whiteCount = sum(sum(final == 1));
HCP = whiteCount/(blackCount+whiteCount);

% Label the nanospheres & calculate the coordinates
[L,N]=bwlabel(final);
stats=regionprops(L,'Centroid');

XY=zeros(N,2);
for n=1:N
    XY(n,:)=[stats(n).Centroid];
end

% Voronoï
[V,C]=voronoin(XY);
[VX, VY] = voronoi(XY(:,1),XY(:,2));
h = plot(VX,VY,'black',XY(:,1),XY(:,2),'black');
set(h(1:end-1),'xliminclude','off','yliminclude','off');

N_neighbor=zeros(N,1);
for n=1:N
    if length(C{n})<7;
        N_neighbor(n)=length(C{n});
    end
end

OR

% Distance
Z = pdist(XY,'euclidean');
Z=squareform(Z);

Nb14=zeros(N,1);
for n=1:N
    L=Z(n,:);
cpt=0;
    for m=1:N
        if L(m)<12,cpt=cpt+1;end
    end
    Nb14(n)=cpt-1;
end
C. Geometry of the NSL

*From Reference [3]*

All labeling refers to Figure C-1.

**Figure C-1**
Geometry of NSL

1. The length of the perpendicular bisector of the largest possible equilateral triangle that fits in the space between the circles, GI is:

\[
GI = \frac{3d}{2} \left( \sqrt{3} - 1 - \frac{1}{\sqrt{3}} \right)
\]  
(C.1)

**Proof:** First, we note that the angle GHI is 60° and angle GHK is 30°, thus GI is 3 times longer than GK because \(\tan(60°)/\tan(30°) = 3\).

\[
GI = 3 \times GK
\]  
(C.2)

Next we note that angle DAK is 30°, and calculate

\[
AK = \frac{AD}{\cos(30°)} = \frac{2r}{\sqrt{3}} = \sqrt{3}r - \frac{r}{\sqrt{3}}
\]  
(C.3)

where \(r = \text{radius AD}\).

\[
GK = AK - r
\]  
(C.4)

From (C.3) and (C.4),
APPENDIX C
Geometry of NSL

\[ \text{GK} = \sqrt{3}r - \frac{r}{\sqrt{3}} - r = \frac{d}{2} \cdot \left( \sqrt{3} - 1 - \frac{1}{\sqrt{3}} \right) \quad \text{(C.5)} \]

where \( 2r = d \) (diameter). Therefore from (C.2) and (C.5),

\[ \text{GI} = \frac{3d}{2} \cdot \left( \sqrt{3} - 1 - \frac{1}{\sqrt{3}} \right) \]

QED.

2. The length of the perpendicular bisector GE is:

\[ \text{GE} = \text{AE} - \text{AG} = \sqrt{3} r - r \]

GE = AE - AG = \sqrt{3} r - r.

Proof:

\[ \text{GE} = \text{AE} - \text{AG} = \text{AE} - r. \quad \text{(C.6)} \]

\[ \text{AE} = \text{AB} \cos(30^\circ) = \sqrt{3} r. \quad \text{(C.7)} \]

Therefore,

\[ \text{GE} = \sqrt{3} r - r. \quad \text{QED.} \]

Note, the length produced during a real deposition depends on the deposition conditions and any subsequent annealing.
D. Shape anisotropy

From References [4, 5]

Although most materials show some magnetocrystalline anisotropy, a polycrystalline sample with no preferred orientation of its grains will have no overall crystalline anisotropy. However, only if the sample is exactly spherical will the same field magnetize it to the same extent in every direction. If the sample is not spherical, then it will be easier to magnetize it along a long axis. This phenomenon is known as shape anisotropy.

In order to understand the origin of shape anisotropy, we first need to introduce the concept of the demagnetizing field.

Demagnetizing field

The concept of a demagnetizing field is confusing, and we will introduce it in a rather qualitative way from the viewpoint of magnetic poles. Let’s suppose that our prolate spheroid from Figure D - 1 has been magnetized by a magnetic field applied from right to left. This results in a north pole at the left end of the prolate spheroid and a south pole at the right end. By definition, the lines of H radiate from the North Pole and end at the South Pole, resulting in the pattern of field lines shown in Figure E - 1. We see from the figure that the field inside the sample points from left to right – that is, in the opposite direction to the applied external field! This internal field tends to demagnetize the magnet, and so we call it the demagnetizing field, $H_d$.

![Figure D - 1](image)

The demagnetizing field is created by the magnetization of the sample, and in fact the size of the demagnetizing field is directly proportional to the size of the magnetization.

We write

$$ H_d = N_d M $$  \hspace{1cm} (D.1)

where $N_d$ is called the demagnetizing factor, and is determined by the shape of the sample.
The value of $N_d$ depends mainly on the shape of the body (Figure D - 2).

The sum of the demagnetizing factors along the three orthogonal axes of an ellipsoid is a constant:

$$N_a + N_b + N_c = 4\pi \text{ (cgs)} \quad (D.2)$$

$$N_a + N_b + N_c = 1 \text{ (SI)} \quad (D.3)$$

For a sphere, the three demagnetizing factors must be equal, so

$$N_{\text{sphere}} = \frac{4\pi}{3} \text{ (cgs)} \text{ or } N_{\text{sphere}} = \frac{1}{3} \text{ (SI)}$$

The general ellipsoid has three unequal axes $2a$, $2b$, $2c$, and a section perpendicular to any axis is an ellipse (Figure E - 2). Of greater practical interest is the ellipsoid of revolution, or spheroid. A prolate spheroid is formed by rotating an ellipse about its major axis $2c$; then $a = b < c$, and the resulting solid is cigar-shaped. Rotation about the minor axis $2a$ results in the disk-shaped oblate spheroid, with $a < b = c$. Although we won’t go into the details here, $N_d$ can be calculated for different shapes (for details, see [5]). The results of the calculations indicate that, for elongated samples, $N_d$ is smallest along the long axis and largest along the short axis. The anisotropy becomes stronger as the aspect ratio increases, with $N_d \rightarrow 0$ as the distance between the poles $\rightarrow \infty$.

Moreover, the effective field acting inside the material, $H_{\text{eff}}$, is smaller than the applied field by an amount equal to the demagnetizing field, i.e.

$$H_{\text{eff}} = H_{\text{applied}} - H_d \quad (D.4)$$

So along the long axis, where $N_d$ is small,

$$H_{\text{eff}} = H_{\text{applied}} - N_dM \cong H_{\text{applied}} \quad (D.5)$$

and most of the applied field goes into magnetizing the sample.
By contrast, along the short axis $N_d$ is large, so

$$H_{\text{eff}} = H_{\text{applied}} - N_d M \ll H_{\text{applied}}$$ (D.6)

and so most of the applied field goes into overcoming the demagnetizing field.

As a consequence it is easier to magnetize the sample along the long axis.

Demagnetizing factors can be very important, and a high field is required to magnetize a sample with a large demagnetizing factor, even if the material has a large susceptibility.
E. MOKE microscopy

*From References [6, 7]*

The principle of the layout is that of polarization microscopy in reflection. The sample is illuminated uniformly by light from the source, focused on the aperture diaphragm, which is itself coupled to the entrance pupil of the microscope objective. The light from the sample is collected by the microscope objective and is focused on the CCD camera.

![Diagram of MOKE microscopy setup](image)

**Figure E - 1**
Scheme of set-up for MOKE microscopy

- **Light source:** it is a white lamp or a LED.
- **Polarizer and analyzer:** The effect of the polarizer as the analyzer on an electric field is to select the component parallel to its axis of transparency.
- **Beam splitter:** It reflects part of light as a semi-transparent and lets the rest.
- **Coil:** it creates a magnetic field.
- **CCD:** it measures light intensity reflected by the sample.
References